2012 John O'Bryan Mathematical Competition
Questions for the Two-Person Speed Event

1. From the four numbers 6, 15, 42, and 51, one number is selected at random. Find the probability that the number selected has exactly two prime positive factors. Express your answer as a common fraction reduced to lowest terms.

2. Eight times an odd integer x is 33 more than the next largest odd integer. A rectangle has perimeter 64 and the longer side is 4 units greater than the shorter side. Find the sum of x and the length of one of the shorter sides of the rectangle.

3. For all real numbers a and b, let $a \otimes b = (a-5)(b+3)$. Let k be the number of sides of a convex polygon whose sum of the degree measures of its interior angles is 2160. Find the value of $(7 \otimes 20) + k$.

4. Let k be the common ratio of an infinite geometric progression with 1^{st} term of 15 and sum of 25. Let w be the common ratio of an infinite geometric progression with 1^{st} term of 63 and sum of 81. Find the value of $(k + w)$. Express your answer as a common fraction reduced to lowest terms.

5. Let k be the probability of drawing 5 hearts when 5 cards are selected at random (without replacement) from 13 hearts and 3 spades. Let S be the sum of all distinct positive integral multiples of 3 that are less than 56. Find the smallest integer that is greater than (kS).

6. The equation of a line whose x-intercept is 4 and which passes through (10,12) can be expressed in the form $y = mx + b$. The inverse of $y = 2x + 3$ can be expressed in the form $y = kx + w$. Find the value of $(m+b+k+w)$.

7. The lengths of all sides of a **scalene** triangle are integers. Two of the sides have lengths of 8 and 11. Let k be the possible number of non-congruent scalene triangles that meet these criteria. Let $A = \{25, 68, 91, 73, 62, 168, w\}$ and let the arithmetic mean (average) of A be 79. Let m be the median of the set A. Find the sum of k, m, and w.

8. Let x be the smallest positive integer such that $\cot(x^\circ) < 0$ and $\cot(x^\circ) = -\cot(-x^\circ)$. Let y be the largest real value for which $y = -2x^2 + 44x - 11$. Find $(y - x)$.

9. (T1) The second term of an arithmetic sequence is 13 and the eighteenth term of this arithmetic sequence is 41. Find the twenty-ninth term. Express your answer as a decimal.

10. (T2) The area of a rectangle is 168 square units, and a diagonal of the rectangle has length 25 units. Find the number of units in the perimeter of this rectangle.

Name: ______ANSWERS______ Team Code: _______

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Answers for the Two-Person Speed Event

Note: **All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value; however ties for individual awards will be broken based on problem difficulty.**

1. 3 / 4 — Must be this fraction

2. 19

3. 60

4. 28 / 45 — Must be this fraction

5. 152

6. -7

7. 147

8. 140

T1. 60.25 — Must be this decimal

T2. 62

This competition consists of eight competitive rounds. Correct answers will receive the following scores:

1st: 7 points
2nd: 5 points
All Others: 3 points

There is a three minute time limit on each round. You may submit only one answer each round. To submit your answer, fold this sheet **lengthwise** and hold it high in the air so that a proctor may check your answer.