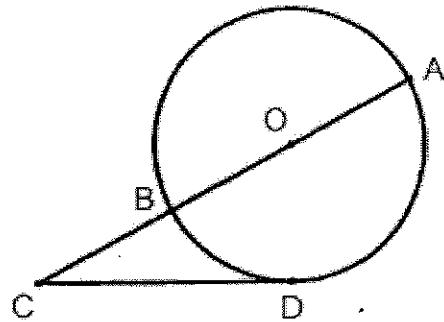


2014 John O'Bryan Mathematical Competition
Junior-Senior Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

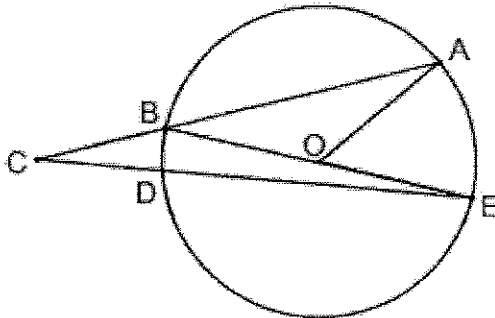
- Find the sum of all distinct values of x such that $\left[\log_k(x^2)\right](\log_{12} k) = 2$.
- Let $i = \sqrt{-1}$. Then $-2i^2 + (\sqrt{-4})(\sqrt{4}) - (\sqrt{-3})(\sqrt{-3}) - 2i^5 = a + bi$, where a and b are real numbers. Find the value of $(3a + 2b)$.
- If x is an integer, find the sum of all distinct values of x such that $\frac{x-4}{x-9} - 3 \geq 0$.

- In the diagram, A , B , and D lie on the circle with center O , \overline{CD} is tangent to the circle at D , and C , B , O , and A are collinear. If \overline{AB} is a diameter, $CD = 12$, and $CB = 4$, find the area of $\triangle COD$.



- The three vertices of a triangle are $(4, 0)$, $(1, 3)$, and $(1, 6)$. Find the smallest possible value of the sum of the squares of the distances of a point to the three vertices of the triangle.
- If $\csc(\theta) = \frac{13}{5}$ and $\tan(\theta) < 0$, find the value of $\cos(\theta)$. Give your answer as a fully reduced fraction.
- A bag contains exactly 3 marbles – 1 red, 1 white, and 1 blue. A girl draws a marble at random, replaces the marble, and continues to draw in this fashion. Find the probability that, in 6 draws, she has drawn at least 1 marble of each color. Give your answer as a fully reduced fraction.

- Points A , B , D , and E lie on the circle with center at O . Point O lies on diameter \overline{BE} . Point B lies on \overline{AC} and point D lies on \overline{CE} . If $\angle AOB = 102^\circ$ and $\angle BEC = 32^\circ$, find the degree measure of $\angle ACE$.



- Shane borrows \$3,375 from a friend. He arranges to pay back \$40 the first month, \$45 the second month, \$50 the third month, and so on by paying \$5 more each month than he did the preceding month. If no interest is charged, how many months will it take Shane to repay the loan? (Note: Shane makes the payments at the end of each month).

10. Find the value of $\log_{27} \left(9 \left(\frac{1}{27} \right)^{-2} \right)$. Give your answer as a fully reduced **improper** fraction.
11. Find the eighth term of an arithmetic progression whose first term is 3 and whose 31st term is 73. Give your answer as a fully reduced **improper** fraction.
12. Suppose that $\frac{8!}{3!k!} = 56$. Find the value of k .
13. When 1, 2, 3, 4, and 5 are substituted for x in a polynomial expression for x , the results are, respectively, 1, 24, 61, 112, and 177. If $P(x)$ is the polynomial expression of lowest degree with integral coefficients satisfying the given, find $P(51)$.
14. One member of the 4 members of the set $\{169, 224, \sqrt{3^2 + 4^2}, 81 + 19\}$ is selected at random. Find the probability that the number selected is the square of an integer. Give your answer as a fully reduced fraction.
15. Jordan plans to invest \$5,000 in a savings account that pays an annual percentage rate of 7% and is compounded quarterly. After the interest is credited at the end of one year, find the value of Jordan's investment. Round your answer to **the nearest dollar**, and express your answer as that whole number.
16. How many non-congruent triangles ABC are possible if $\angle B$ is a given acute angle, $BC = 12$ and $AC = 15$?
17. Find the length of the major axis of the ellipse whose equation is $\frac{x^2}{49} + \frac{y^2}{25} = 1$.
18. A sector with a 60° central angle is cut out of a circle with a radius of 6, and the remaining 300° sector is folded to form a right circular cone. Find the volume of this right circular cone. Express your answer as a **decimal** rounded to the nearest hundredth.
19. A line whose equation is $40y = 9x + k$ is parallel to a line whose equation is $y = \frac{9}{40}x + 17$. If $k > 1000$, and if the distance between the two lines is 40, find the value of k .
20. Let $f(x) = x^3 - 13.5x^2 + 42x + 39$. The set of values for k such that $f(x) - k$ will have three distinct real zeroes is $\{k : w < k < p\}$. Find the value of $(w + p)$. Express your answer as an **exact decimal**.

Name: _____

Team Code: _____

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1. _____

11. _____

2. _____

12. _____

3. _____

13. _____

4. _____

14. _____

5. _____

15. _____

6. _____

16. _____

7. _____

17. _____

8. _____

18. _____

9. _____

19. _____

10. _____

20. _____

Name: ANSWERS

Team Code: _____

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Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value (1 point).

1. 0 or zero

2. 19

3. 21

4. 96

5. 24

6. $\frac{12}{13}$ Must be this fraction.

7. $\frac{20}{27}$ Must be this fraction.

8. 7

9. 30

10. $\frac{8}{3}$ Must be this improper fraction.

11. $\frac{58}{3}$ Must be this improper fraction.

12. 5

13. 18301

14. $\frac{1}{2}$ Must be this fraction.

15. 5359

16. 1 or one

17. 14

18. 86.83 Must be this decimal.

19. 2320

20. 91.5