2011 John O'Bryan Mathematical Competition
Freshman-Sophomore Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value; however ties for individual awards will be broken based on problem difficulty (#11-20 are considered more difficult and will be used to break ties).

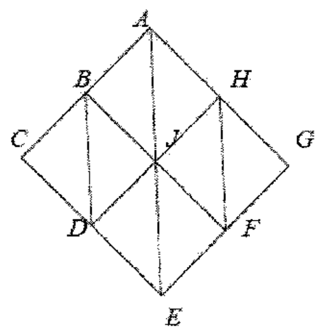
1. A convex polygon has 19 sides. Find the sum of the degree measures of the interior angles of this polygon.

2. If $\left(x^2 - y^2\right) = 0$ and $x + y = 17.534$, find the value of $x - y$.

3. Two circles are constructed such that one has radius of length 8 and the other has circumference of length 64π. The area of the larger circle is k times the area of the smaller circle. Find the value of k.

4. Casey invested part of \$100,000 in bonds that earned interest at 6% annual percentage rate. The rest he invested in certificates that earned interest at 5% annual percentage rate. At the end of one year, his interest income from these totaled \$5320. Find the number of dollars Casey invested in certificates.

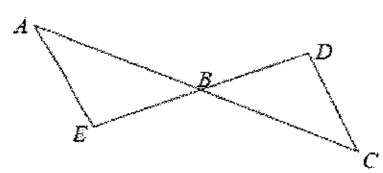
5. As shown in the picture at the right, the following sets of points are collinear: ABC, CDE, EGF, GHA, BJF, DJH, and AJE. How many different paths from A to E are possible if only downward movements are allowed (for example, one may travel from B to C, D, or J, but not from B to A)?



6. A bag of shapes contains the following polygons: a square, a triangle, a pentagon, and a hexagon. Two polygons are selected from this bag at random and without replacement. Find the probability that the total number of sides for the selected polygons is at least 9.

7. A man bought a 36-ounce Pepsi. On the first day he drank 1 ounce from the container and then refilled the remainder of the container with Coke. On the second day, he drank 2 ounces from the container and refilled the remainder of the container with Coke. On the third day, he drank 3 ounces from the container and refilled the remainder of the container with Coke. This procedure was continued for succeeding days until the container was empty. Find the total number of ounces of coke the man drank during this process.

8. In the diagram at the right, $\overline{AC}$ and $\overline{ED}$ intersect at B. Additionally, $\overline{AE} \perp \overline{ED}$, $\overline{CD} \perp \overline{ED}$, $AE = 5$, $DC = 7$, and $AC = 30$. Find AB. Express your answer as a decimal.



9. How many integers between 1 and 100 (including 1 and 100) are neither the square nor cube of an integer?

10. The number of inches in the length of the minor arc intercepted by a chord whose length is $\sqrt{1200}$ inches is $k\pi$ inches. If the radius of this circle is 20 inches, find the value of *k*. Express your answer as an improper fraction reduced to lowest terms.

11. Let *x*, *y*, and *z* represent integers greater than 1. If $\left(x^7\right)^y = x^7 x^z$, find the sum of the smallest two distinct values of $|z - y|$.

12. Two integers are formed by writing the same two digits in reverse order. The sum of the two integers is 22 times the difference between the digits. Also, the difference between the squares of the two integers is *m* where $m > 1000$. If none of the digits of *m* are the same as any digits used to form the original two integers, find the larger of the two original integers.

13. A triangle has sides of lengths of 34, 34, and 32. Find the radius of the circle inscribed in that triangle. Express your answer as a decimal.

14. Let *x* and *y* be positive integers such that the average of $5x$ and $7y$ is 60. Find the sum of all distinct possible values of *x*.

15. A right triangle has sides of integer length. The difference between the length of the hypotenuse and the length of the longer leg is the same as the difference between the length of the longer leg and the length of the shorter leg. If the area of the triangle is 600, find the length of the radius of the circumscribed circle.

16. A truck was originally priced at $25,000. It's price was reduced by *r*% to *x* dollars. Then the price of *x* dollars was again reduced by *k*% to $16170. If both *r* and *k* are positive integers with $r < k$, find the smallest possible value of $2k$.

17. *ABCD* and *EFGH* are rectangles such that the lengths of all sides and diagonals are integers. The area of ABCD is $\frac{7}{6}$ of the area of EFGH. If the length diagonal *HF* is 41, find the sum of all possible distinct lengths of diagonal $\overline{DB}$.

18. Twice the sum of the squares of the roots for $x^2 - 70x + k = 0$ is added to the product of the roots and the result is 6632. Find the smaller of the two roots for *x*.

19. *AEC* is an acute triangle, with *B* lying on $\overline{AC}$ and *D* lying on $\overline{CE}$. $\overline{AD}$ and $\overline{BE}$ intersect at *F*. If $DC = 5(DE)$ and $AB{:}BC = 4{:}5$, then the ratio of the area of triangle *ABF* to the area of quadrilateral *BCDF* is $k : w$ where *k* and *w* are positive integers. Find the smallest possible value of $k + w$.

20. The number *x* is five more than the product of two consecutive positive integers. The number *x* is also an integral multiple of both 55 and 121. Let the smaller of the two consecutive positive integers be represented by *k*. Find the sum of all distinct values of *k* if $k < 157$.

Name: ______ANSWERS______ Team Code: _______

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1. 3060
2. 0
3. 16
4. 68000
5. 13
6. 2/3
7. 630
8. 12.5
9. 88
10. 40/3
11. 16
12. 93
13. 9.6
14. 30
15. 25
16. 46
17. 66
18. 22
19. 87
20. 169